

Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- [5]

[illegible]

2 Find the Maclaurin's series for $e^{1+x^2} + e^{1-x}$ up to and including the term in x^2 . [4]

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The lines are thin and light gray, set against a plain white background. There are no margins, text, or other markings on the page.

3 It is given that

$$x = \sin^{-1} t \quad \text{and} \quad y = t \cos^{-1} t, \quad \text{for } 0 \leq t < 1.$$

(a) Show that $\frac{dy}{dx} = -t + \sqrt{1-t^2} \cos^{-1} t$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) Find $\frac{d^2y}{dx^2}$ in terms of t .

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

4 It is given that, for $n \geq 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \, dx$.

(a) Show that, for $n \geq 2$,

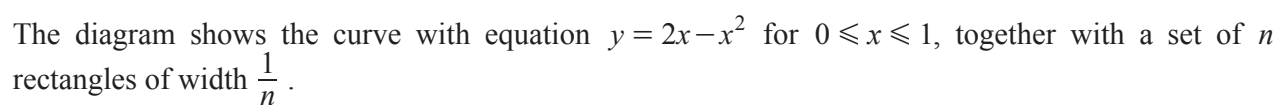
$$(n-1)I_n = \left(\frac{3}{5}\right)^{n-2} \left(\frac{4}{5}\right) + (n-2)I_{n-2}. \quad [5]$$

[You may use the result that $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$.]

[illegible]

[3]

[illegible]



- $$U_n = \left(1 + \frac{1}{n}\right) \left(\frac{2}{3} - \frac{1}{6n}\right). \quad [5]$$

[illegible]

- (b)** Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 (2x - x^2) dx$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

- 6 (a) Show that $(\cosh x + \sinh x)^{\frac{1}{2}} = e^{\frac{1}{2}x}$. [2]

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- (b) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5(\cosh x + \sinh x)^{\frac{1}{2}},$$

given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$. [10]

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- 7 (a) Use the substitution $u = 1 + x^2$ to find

$$\int \frac{x}{\sqrt{1+x^2}} dx. \quad [2]$$

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- (b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \sinh^{-1} x,$$

given that $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$. [10]

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- 8 (a) Find the set of values of a for which the system of equations

$$6x + ay = 3,$$

$$2x - y = 1,$$

$$x + 5y + 4z = 2$$

has a unique solution.

[2]

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- (b) Show that the system of equations in part (a) is consistent for all values of a .

[3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 5 & 4 \end{pmatrix}.$$

- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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